NONLINEAR PROBLEMS IN THERMAL CONDUCTION*

Reviewed by B. M. Smol'skii

These two monographs deal with methods of solving some of the most complicated problems in thermal conduction (nonlinear ones), and also with simulation results. The interest in this area is increasing rapidly, in part because of increasingly stringent specifications for the accuracy of calculations on temperature patterns in structures over a wide range of temperatures (from cryogenic temperatures up to the melting point).

Both books are among the first to appear in their areas in any country. They reflect the ongoing tendency to employ simulation techniques in practical heat-engineering calculations, which began with linear problems and problems with variable coefficients, particularly in the monographs by Lykov and by Carslaw and Jaeger.

The first of these volumes is essentially a handbook on methods of solving thermal-conduction problems when the thermophysical characteristics of the materials are functions of temperature, as are the outputs of the surface and internal heat sources and the coordinates of the boundaries (nonlinearities of types I-III).

A classification is given of the mathematical models, types of nonlinearity, and types of problem that have been solved and (especially) research methods. A special section deals with what is always an important aspect, viz., choice of method of solution and means of deriving numerical results. Although the use of numerical methods is very much traditional, whether digital or analog techniques are employed, the volumes give an objective characterization of the pros and cons of the various approximate analytical and numerical methods that may be used to handle nonlinear problems with nonlinearities of types I, II, and III. The volume on methods for nonlinear thermal conduction (diffusion) gives a detailed bibliography (over 900 Soviet and other papers) on methods and results in this area. Each chapter has its own list of literature cited, which enables the reader to evaluate the utility of methods and which provides information on work in other areas. The volume dated 1975 deals in detail with principles, and some interesting examples are given of solutions to nonlinear problems involving various types of nonlinearity, which are handled by variational, iterative, and integral techniques (in particular, all weighted-residue methods are described); information is also given on other types of equations, perturbation equations, finite-difference methods, direct techniques, etc. One chapter is devoted to linearization and an analysis of the resulting errors. The first volume demonstrates the utility of the methods on examples, while the second presents a detailed analysis of various numerical solutions to nonlinear problems. In particular, a study is made of the effects of numerous factors on the errors of solutions to nonlinear problems obtained by numerical methods. Particular interest is attached to the chapters dealing with the solution of nonlinear inverse problems in thermal conduction. Conjugate problems in nonstationary heat transfer and the related inverse problems represent a growth point in this area, where researchers in the theory of thermal conduction have obtained very significant results in recent years. Considerable interest attaches also to the sections dealing with the analysis of errors arising from linearization (the partial-linearization paradox), which present original results on the solution of nonstationary problems for heat shields; in addition, there are sections dealing with oscillations in numerical solutions, the effects of nonlinearities on the choice of time and space steps (violation of Runge's principle), etc. The numerous examples given in both volumes of the solution of nonlinear heat-conduction problems are of considerable practical value, since nonlinear problems are essentially of concrete nature, and only an analysis of solutions to concrete problems can provide conclusions of any value on the effects of nonlinearities. Particular note may be made of the highly successful mode of exposition, with its

*L. A. Kozdoba, Methods of Solving Nonlinear Problems in Thermal Conduction [in Russian], Nauka, Moscow (1975); L. A. Kozdoba, Solution of Nonlinear Problems in Thermal Conduction [in Russian], Naukova Dumka, Kiev (1976).

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clear and explicit statement of the essence of each problem, which is accompanied by a reasonably detailed presentation not overloaded with minor aspects. This form of exposition is of value to those requiring to extend their knowledge or to apply it to particular problems as well as to those interested in nonlinearities generally.

The critical exposition enables the reader to avoid repeating some typical errors, while the numerous and complicated researches performed by the author provide a basis for rapid acquaintance with the current state of the art in the field of analytical and numerical methods of solving nonlinear problems.

There is no doubt that these volumes will accelerate progress in the nonlinear theory of thermal conduction, which is still an underdeveloped area.

Although the volumes present a large variety of problems from various areas in heat engineering, none of the examples reflects the effects of nonlinearities of types I, II, and III on the behavior of the solutions for problems in two and three dimensions (only brief indications are given), nor is there any analysis of studies on other methods (apart from trial and error) in the solution of nonlinear inverse problems in thermal conduction and diffusion. Further, the volumes lack sections on the solution of nonlinear conjugate problems, particularly nonstationary ones. There is also no universally agreed view on which methods should be called direct. Here a comment made by Academician Sobolev in 1947 should be borne in mind: "The name direct should be given to methods of approximate solution of differential and integral equations that reduce such problems to finite systems of algebraic equations" (see S. G. Mikhlin, Variational Methods in Mathematical Physics [in Russian], Nauka, Moscow (1970), p. 11).

On the whole, the material in both volumes is presented at a high theoretical level and is well illustrated with figures, graphs, and interesting practical applications of the various methods. The first volume is particularly good in this respect, and it is unfortunate that the edition was small and the book has already become a bibliographic rarity.

It is suggested that any fresh edition should combine the material from both volumes, while the reference section should be supplemented with data from later publications, particularly on the use of analytical and numerical methods in mathematical handling of nonlinear problems in heat and mass transfer.